Mathematics
Class – 9th

NCERT Solutions for Class IX Maths: Chapter 1 – Number Systems

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Exercise 1.1
1. Is zero a rational number? Can you write it in the form p/q, where$p$ and q are integers and $q \neq 0$?
Answer
Yes. Zero is a rational number as it can be represented as 0/1 or 0/2.

2. Find six rational numbers between 3 and 4.

Answer
There are infinite rational numbers in between 3 and 4.
3 and 4 can be represented as 24/8 and 32/8 respectively.
Therefore, six rational numbers between 3 and 4 are
25/8, 26/8, 27/8, 28/8, 29/8, 30/8.

3. Find five rational numbers between 3/5 and 4/5.

Answer
There are infinite rational numbers in between 3/5 and 4/5
3/5 = 3×6/5×6 = 18/30
4/5 = 4×6/5×6 = 24/30
Therefore, five rational numbers between 3/5 and 4/5 are
19/30, 20/30, 21/30, 22/30, 23/30.

4. State whether the following statements are true or false. Give reasons for your answers.
(i) Every natural number is a whole number.
► True, since the collection of whole numbers contains all natural numbers.

(ii) Every integer is a whole number.
► False, as integers may be negative but whole numbers are always positive.

(iii) Every rational number is a whole number.

► False, as rational numbers may be fractional but whole numbers may not be.
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Exercise 1.2
1. State whether the following statements are true or false. Justify your answers.
(i) Every irrational number is a real number.

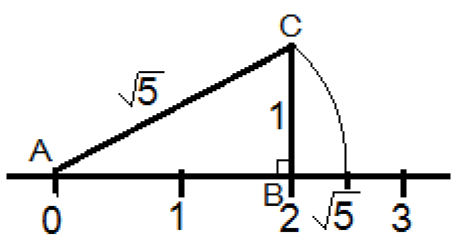
► True, since the collection of real numbers is made up of rational and irrational numbers.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.
► False, since positive number cannot be expressed as square roots.
(iii) Every real number is an irrational number.
► False, as real numbers include both rational and irrational numbers. Therefore, every real number cannot be an irrational number.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.
Answer
No, the square roots of all positive integers are not irrational. For example $\sqrt{4} = 2$.

3. Show how $\sqrt{5}$ can be represented on the number line.

Answer
Step 1: Let AB be a line of length 2 unit on number line.
Step 2: At B, draw a perpendicular line BC of length 1 unit. Join CA.
Step 3: Now, ABC is a right angled triangle. Applying Pythagoras theorem,
$AB^2 + BC^2 = CA^2$
$\Rightarrow 2^2 + 1^2 = CA^2$
$\Rightarrow CA^2 = 5$
$\Rightarrow CA = \sqrt{5}$
Thus, CA is a line of length $\sqrt{5}$ unit.
Step 4: Taking CA as a radius and A as a centre draw an arc touching the number line. The point at which number line get intersected by arc is at $\sqrt{5}$ distance from 0 because it is a radius of the circle whose centre was A.
Thus, $\sqrt{5}$ is represented on the number line as shown in the figure.



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Exercise 1.3
1. Write the following in decimal form and say what kind of decimal expansion each has:
(i) 36/100
= 0.36 (Terminating)
(ii) 1/11
0.09090909… = 0.9 (Non terminating repeating)

(iii) $4\frac{1}{8}$
= 33/8 = 4.125 (Terminating)

(iv) 3/13
= 0.230769230769… = 0.230769 (Non terminating repeating)

(v) 2/11
= 0.181818181818… = 0.18 (Non terminating repeating)

(vi) 329/400
= 0.8225 (Terminating)

2. You know that 1/7 = 0.142857.Can you predict what the decimal expansion of 2/7, 3/7, 4/7, 5/7, 6/7 are without actually doing the long division? If so, how?
[Hint: Study the remainders while finding the value of 1/7 carefully.]

Answer
Yes. We can be done this by:

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$$

$$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$$

3. Express the following in the form p/q where p and q are integers and $q \neq 0$.
(i) 0.6

(ii) 0.47

(iii) 0.001

Answer
(i) 0.6 = 0.666…
Let x = 0.666…
10x = 6.666…
10x = 6 + x
9x = 6
x = 2/3
(ii) 0.47 = 0.4777…
= 4/10 + 0.777/10
Let x = 0.777…
10x = 7.777…
10x = 7 + x
x = 7/9
4/10 + 0.777…/10 = 4/10 + 7/90

= 36/90 +7/90 = 43/90

(iii) 0.001 = 0.001001…

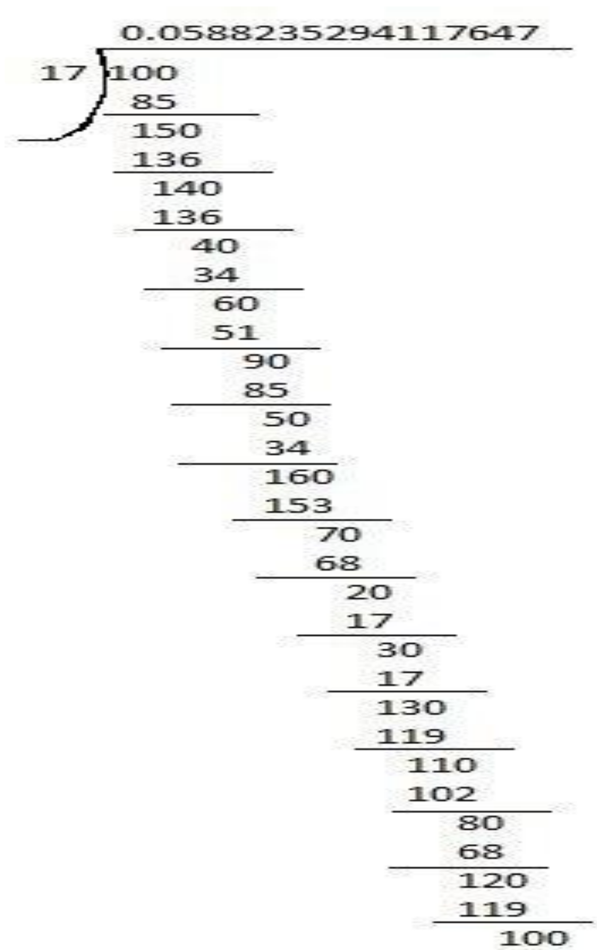
Let x = 0.001001…
1000x = 1.001001…
1000x = 1 + x
999x = 1
x = 1/999
4. Express 0.99999…in the form p/q. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Answer

Let x = 0.9999…
10x = 9.9999…
10x = 9 + x
9x = 9
x = 1
The difference between 1 and 0.999999 is 0.000001 which is negligible. Thus, 0.999 is too much near 1, Therefore, the 1 as answer can be justified.
5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of 1/17? Perform the division to check your answer.

Answer

1/17 = 0.0588235294117647
There are 16 digits in the repeating block of the decimal expansion of 1/17.
Division Check:



= 0.0588235294117647

6. Look at several examples of rational numbers in the form p/q ($q \neq 0$) where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Answer

We observe that when q is 2, 4, 5, 8, 10… then the decimal expansion is terminating. For example:

1/2 = 0.5, denominator $q = 2^1$

7/8 = 0.875, denominator $q = 2^3$

4/5 = 0.8, denominator $q = 5^1$

We can observed that terminating decimal may be obtained in the situation where prime factorisation of the denominator of the given fractions has the power of 2 only or 5 only or both.

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Answer

Three numbers whose decimal expansions are non-terminating non-recurring are:

0.303003000300003…

0.505005000500005…

0.7207200720007200007200000…

8. Find three different irrational numbers between the rational numbers 5/7 and 9/11.

Answer

5/7 = 0.714285

9/11 = 0.81

Three different irrational numbers are:

0.73073007300073000073…

0.75075007300075000075…

0.76076007600076000076…

9. Classify the following numbers as rational or irrational:

(i) √23

(ii) √225

(iii) 0.3796

(iv) 7.478478

(v) 1.101001000100001…

Answer
(i) √23 = 4.79583152331…

Since the number is non-terminating non-recurring therefore, it is an irrational number.

(ii) √225 = 15 = 15/1
Since the number is rational number as it can represented in p/qform.
(iii) 0.3796

Since the number is terminating therefore, it is an rational number.

(iv) 7.478478 = 7.478

Since the this number is non-terminating recurring, therefore, it is a rational number.

(v) 1.101001000100001…

Since the number is non-terminating non-repeating, therefore, it is an irrational number.

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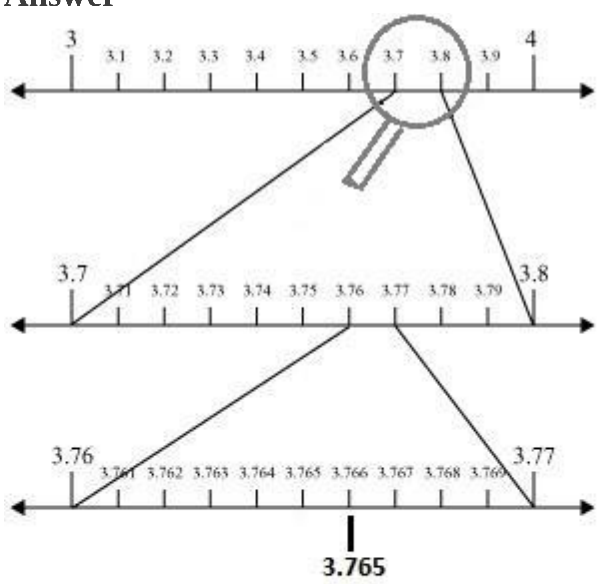
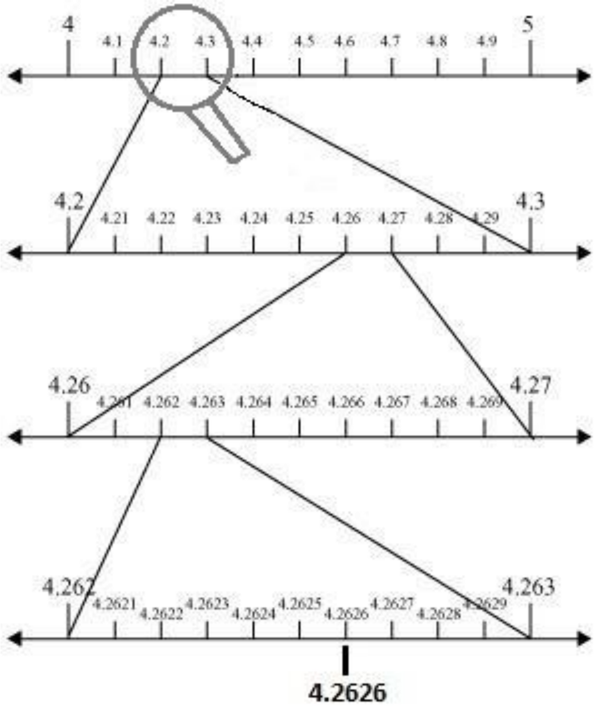
Exercises 1.4
1. Visualise 3.765 on the number line using successive magnification.

Answer



2. Visualise 4.26 on the number line, up to 4 decimal places.

Answer
4.26 = 4.2626



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Exercise 1.5

1. Classify the following numbers as rational or irrational:

(i) 2 – √5

(ii) (3 + √23) – √23

(iii) 2√7/7√7

(iv) 1/√2

(v) 2π

Answer

(i) 2 – √5 = 2 – 2.2360679… = – 0.2360679…
Since the number is is non-terminating non-recurring therefore, it is an irrational number.

(ii) (3 + √23) – √23 = 3 + √23 – √23 = 3 = 3/1
Since the number is rational number as it can represented in p/q form.
(iii) 2√7/7√7 = 2/7
Since the number is rational number as it can represented in p/q form.
(iv) 1/√2 = √2/2 = 0.7071067811…
Since the number is is non-terminating non-recurring therefore, it is an irrational number.

(v) 2π = 2 × 3.1415… = 6.2830…
Since the number is is non-terminating non-recurring therefore, it is an irrational number.

2. Simplify each of the following expressions:

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$

(ii) $(3 + \sqrt{3})(3 - \sqrt{3})$

(iii) $(\sqrt{5} + \sqrt{2})^2$
(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$
Answer
(i) $(3 + \sqrt{3})(2 + \sqrt{2})$

$\Rightarrow 3 \times 2 + 2 + \sqrt{3} + 3\sqrt{2} + \sqrt{3} \times \sqrt{2}$

$\Rightarrow 6 + 2\sqrt{3} + 3\sqrt{2} + \sqrt{6}$

(ii) $(3 + \sqrt{3})(3 - \sqrt{3})$ [$\because (a + b)(a - b) = a^2 - b^2$]
$\Rightarrow 3^2 - (\sqrt{3})^2$
$\Rightarrow 9 - 3$

$\Rightarrow 6$

(iii) $(\sqrt{5} + \sqrt{2})^2$ [$\because (a + b)^2 = a^2 + b^2 + 2ab$] $\Rightarrow (\sqrt{5})^2 + (\sqrt{2})^2 + 2 \times \sqrt{5} \times \sqrt{2}$
$\Rightarrow 5 + 2 + 2 \times \sqrt{5} \times 2$
$\Rightarrow 7 + 2\sqrt{10}$

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$ [$\because (a + b)(a - b) = a^2 - b^2$]
$\Rightarrow (\sqrt{5})^2 - (\sqrt{2})^2$
$\Rightarrow 5 - 2$

$\Rightarrow 3$

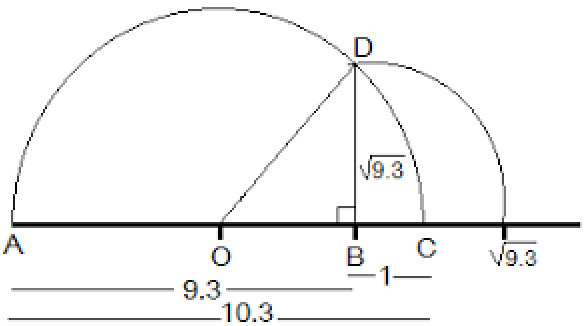
3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, π = c/d. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer
There is no contradiction. When we measure a value with a scale, we only obtain an approximate value. We never obtain an exact value. Therefore, we may not realise that either c or d is irrational. The value of π is almost equal to 22/7 or 3.142857…

4. Represent $\sqrt{9.3}$ on the number line.

Answer
Step 1: Draw a line segment of unit 9.3. Extend it to C so that BC is of 1 unit.
Step 2: Now, AC = 10.3 units. Find the centre of AC and name it as O.
Step 3: Draw a semi circle with radius OC and centre O.
Step 4: Draw a perpendicular line BD to AC at point B which intersect the semicircle at D. Also,

Join OD.
Step 5: Now, OBD is a right angled triangle.
Here, OD = 10.3/2 (radius of semi circle), OC = 10.3/2, BC = 1
OB = OC – BC = (10.3/2) – 1 = 8.3/2
Using Pythagoras theorem,
$OD^2 = BD^2 + OB^2$
⇒ $(10.3/2)^2 = BD2 + (8.3/2)^2$
⇒ $BD^2 = (10.3/2)^2 – (8.3/2)^2$
⇒ $BD^2 = (10.3/2 – 8.3/2) (10.3/2 + 8.3/2)$
⇒ $BD^2 = 9.3$
⇒ $BD^2 = \sqrt{9.3}$
Thus, the length of BD is √9.3.
Step 6: Taking BD as radius and B as centre draw an arc which touches the line segment. The point where it touches the line segment is at a distance of √9.3 from O as shown in the figure.



5. Rationalise the denominators of the following:

(i) 1/√7

(ii) 1/√7-√6

(iii) 1/√5+√2

(iv) 1/√7-2

Answer

(i) $\frac{1}{\sqrt{7}} = \frac{1 \times \sqrt{7}}{1 \times \sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1\ (\sqrt{7}+\sqrt{6})}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$

$= \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2 - (\sqrt{6})^2} = \frac{\sqrt{7}+\sqrt{6}}{1} = \sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1\ (\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$

$= \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2 - (\sqrt{2})^2} = \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$

(iv) $\frac{1}{\sqrt{7}-2} = \frac{1\ (\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)}$

$= \frac{\sqrt{7}+2}{(\sqrt{7})^2 - (2)^2} = \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$

Exercise 1.6

1. Find:

(i) $64^{1/2}$
(ii) $32^{1/5}$
(iii) $125^{1/3}$

Answer

(i) $64^{\frac{1}{2}} = (2^6)^{\frac{1}{2}} = 2^{6\times\frac{1}{2}} = 2^3 = 8$

(ii) $32^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = (2)^{5\times\frac{1}{5}} = 2$

(iii) $(125)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5$

2. Find:

(i) $9^{3/2}$
(ii) $32^{2/5}$
(iii) $16^{3/4}$
(iv) $125^{-1/3}$

Answer

(i) $9^{\frac{3}{2}} = (3^2)^{\frac{3}{2}} = 3^{2\times\frac{3}{2}} = 3^3 = 27$

(ii) $32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = (2)^{5\times\frac{2}{5}} = 2^2 = 4$

(iii) $(16)^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^3 = 8$

(iv) $(125)^{\frac{-1}{3}} = \frac{1}{(125)^{\frac{1}{3}}} = \frac{1}{(5^3)^{\frac{1}{3}}} = \frac{1}{5}$

3. Simplify:

(i) $2^{2/3}.2^{1/5}$
(ii) $(1/3^3)^7$
(iii) $11^{1/2}/11^{1/4}$
(iv) $7^{1/2}.8^{1/2}$

Answer

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} = 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$

(ii) $\left(\frac{1}{3}\right)^7 = \frac{1}{3^{3\times7}} = \frac{1}{3^{21}} = 3^{-21}$

(iii) $\frac{11^{\frac{1}{2}}}{11^{4}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}} = 7 \times 8^{\frac{1}{2}} = 56^{\frac{1}{2}}$

NCERT Solutions for Class IX Maths: Chapter 2 – Polynomials

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Exercise 2.1

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 – 3x + 7$
(ii) $y^2 + \sqrt{2}$
(iii) $3\sqrt{t} + t\sqrt{2}$
(iv) $y + 2/y$
(v) $x^{10} + y^3 + t^{50}$

Answer

(i) $4x^2 – 3x + 7$
There is only one variable x with whole number power so this polynomial in one variable.
(ii) $y^2 + \sqrt{2}$
There is only one variable y with whole number power so this polynomial in one variable.
(iii) $3\sqrt{2} + t\sqrt{2}$
There is only one variable t but in $3\sqrt{t}$ power of t is 1/2 which is not a whole number so $3\sqrt{t} + t\sqrt{2}$ is not a polynomial.
(iv) $y + 2/y$
There is only one variable y but $2/y = 2y$-1 so the power is not a whole number so $y + 2/y$ is not a polynomial.
(v) $x^{10} + y^3 + t^{50}$
There are three variable x, y and t and there powers are whole number so this polynomial in three variable.

2. Write the coefficients of x^2 in each of the following:
(i) $2 + x^2 + x$
(ii) $2 – x^2 + x^3$
(iii) $\frac{\pi}{2}x^2 + x$
(iv) $\sqrt{2}x – 1$

Answer

(i) coefficients of x^2 = 1
(ii) coefficients of x^2 = -1
(iii) coefficients of x^2 = π/2
(iv) coefficients of x^2 = 0

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Answer

$3x^{35}+7$ and $4x^{100}$

4. Write the degree of each of the following polynomials:

(i) $5x^3 + 4x^2 + 7x$
(ii) $4 – y^2$
(iii) $5t – \sqrt{7}$
(iv) 3

Answer
(i) $5x^3$ has highest power in the given polynomial which power is 3. Therefore, degree of polynomial is 3.
(ii) $-y^2$ has highest power in the given polynomial which power is 2. Therefore, degree of polynomial is 2.
(iii) $5t$ has highest power in the given polynomial which power is 1. Therefore, degree of polynomial is 1.
(iv) There is no variable in the given polynomial. Therefore, degree of polynomial is 0.

5. Classify the following as linear, quadratic and cubic polynomial:

(i) $x^2 + x$
► Quadratic Polynomial
(ii) $x - x^3$
► Cubic Polynomial
(iii) $y + y^2 + 4$
► Quadratic Polynomial
(iv) $1 + x$
► Linear Polynomial
(v) $3t$
►Linear Polynomial
(vi) r^2
► Quadratic Polynomial

(vii) $7x^3$
► Cubic Polynomial
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Exercise 2.2
1. Find the value of the polynomial at $5x + 4x^2 + 3$ at
(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$
Answer
(i) $p(x) = 5x + 4x^2 + 3$
$p(0) = 5(0) + 4(0)^2 + 3$
$= 3$
(ii) $p(x) = 5x + 4x^2 + 3$
$p(-1) = 5(-1) + 4(-1)^2 + 3$
$= 5 - 4(1) + 3 = -6$
(iii) $p(x) = 5x + 4x^2 + 3$
$p(2) = 5(2) + 4(2)^2 + 3$
$= 10 - 16 + 3 = -3$
2. Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:
(i) $p(y) = y^2 - y + 1$
(ii) $p(t) = 2 + t + 2t^2 - t^3$
(iii) $p(x) = x^3$
(iv) $p(x) = (x - 1)(x + 1)$
Answer
(i) $p(y) = y^2 - y + 1$
$p(0) = (0)^2 - (0) + 1 = 1$

$p(1) = (1)^2 – (1) + 1 = 1$
$p(2) = (2)^2 – (2) + 1 = 3$
(ii) $p(t) = 2 + t + 2t^2 – t^3$
$p(0) = 2 + 0 + 2 (0)^2 – (0)^3 = 2$
$p(1) = 2 + (1) + 2(1)^2 – (1)3$
$= 2 + 1 + 2 – 1 = 4$
$p(2) = 2 + 2 + 2(2)^2 – (2)^3$
$= 2 + 2 + 8 – 8 = 4$
(iii) $p(x) = x^3$
$p(0) = (0)^3 = 0$
$p(1) = (1)^3 = 1$
$p(2) = (2)^3 = 8$
(iv) $p(x) = (x – 1) (x + 1)$
$p(0) = (0 – 1) (0 + 1) = (- 1) (1) = – 1$
$p(1) = (1 – 1) (1 + 1) = 0 (2) = 0$
$p(2) = (2 – 1) (2 + 1) = 1(3) = 3$
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3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1$, $x = -1/3$
(ii) $p(x) = 5x – \pi$, $x = 4/5$
(iii) $p(x) = x^2 – 1$, $x = 1, -1$
(iv) $p(x) = (x + 1) (x – 2)$, $x = -1, 2$
(v) $p(x) = x^2$, $x = 0$
(vi) $p(x) = lx + m, x = -\frac{m}{l}$
(vii) $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$
(viii) $p(x) = 2x + 1$, $x = 1/2$
Answer
(i) If $x = -1/3$ is a zero of polynomial $p(x) = 3x + 1$ then $p(-1/3)$ should be 0.
At, $p(-1/3) = 3(-1/3) + 1 = -1 + 1 = 0$
Therefore, $x = -1/3$ is a zero of polynomial $p(x) = 3x + 1$.
(ii) If $x = 4/5$ is a zero of polynomial $p(x) = 5x – \pi$ then $p(4/5)$ should be 0.
At, $p(4/5) = 5(4/5) – \pi = 4 – \pi$
Therefore, $x = 4/5$ is not a zero of given polynomial $p(x) = 5x – \pi$.
(iii) If $x = 1$ and $x = -1$ are zeroes of polynomial $p(x) = x^2 – 1$, then$p(1)$ and $p(-1)$ should be 0.
At, $p(1) = (1)^2 – 1 = 0$ and
At, $p(-1) = (-1)^2 – 1 = 0$
Hence, $x = 1$ and -1 are zeroes of the polynomial $p(x) = x^2 – 1$.
(iv) If $x = -1$ and $x = 2$ are zeroes of polynomial $p(x) = (x +1) (x – 2)$, then $p(– 1)$ and (2)should be 0.
At, $p(-1) = (-1 + 1) (-1 – 2) = 0 (-3) = 0$, and
At, $p(2) = (2 + 1) (2 – 2) = 3 (0) = 0$
Therefore, $x = -1$ and $x = 2$ are zeroes of the polynomial $p(x) = (x +1) (x – 2)$.

(v) If $x = 0$ is a zero of polynomial $p(x) = x^2$, then $p(0)$ should be zero.
Here, $p(0) = (0)^2 = 0$
Hence, $x = 0$ is a zero of the polynomial $p(x) = x^2$.

(vi) If $x = \frac{-m}{l}$ is a zero of polynomial $p(x) = lx + m$ then $p\left(\frac{-m}{l}\right)$ should be 0.

At, $p\left(\frac{-m}{l}\right) = l\left(\frac{-m}{l}\right) + m = -m + m = 0$

Therefore, $x = \frac{-m}{l}$ is a zero of the polynomial $(x) = lx + m$

(vii) If $x = \frac{-1}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$ are zeroes of polynomial $p(x) = 3x^2 + 1$, then

$p\left(\frac{-1}{\sqrt{3}}\right)$ and $p\left(\frac{2}{\sqrt{3}}\right)$ should be 0.

At, $p\left(\frac{-1}{\sqrt{3}}\right) = 3\left(\frac{-1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0$ and

At, $p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3$

Therefore, $x = \frac{-1}{\sqrt{3}}$ is a zero of polynomial $p(x) = 3x^2 + 1$

but $x = \frac{2}{\sqrt{3}}$ is not a zero of this polyomial.

(viii) If $x = 1/2$ is a zero of polynomial $p(x) = 2x + 1$ then $p(1/2)$ should be 0.
At, $p(1/2) = 2(1/2) + 1 = 1 + 1 = 2$
Therefore, $x = 1/2$ is not a zero of given polynomial $p(x) = 2x + 1$.
4. Find the zero of the polynomial in each of the following cases:
(i) $p(x) = x + 5$
(ii) $p(x) = x – 5$
(iii) $p(x) = 2x + 5$
(iv) $p(x) = 3x – 2$
(v) $p(x) = 3x$
(vi) $p(x) = ax$, $a \neq 0$
(vii) $p(x) = cx + d$, $c \neq 0$, c, are real numbers.
Answer
(i) $p(x) = x + 5$
$p(x) = 0$
$x + 5 = 0$
$x = -5$
Therefore, $x = -5$ is a zero of polynomial $p(x) = x + 5$.
(ii) $p(x) = x – 5$
$p(x) = 0$
$x – 5 = 0$
$x = 5$
Therefore, $x = 5$ is a zero of polynomial $p(x) = x – 5$.
(iii) $p(x) = 2x + 5$
$p(x) = 0$
$2x + 5 = 0$
$2x = -5$
$x = -5/2$
Therefore, $x = -5/2$ is a zero of polynomial $p(x) = 2x + 5$.

(iv) $p(x) = 3x - 2$
$p(x) = 0$
$3x - 2 = 0$
$x = 2/3$
Therefore, $x = 2/3$ is a zero of polynomial $p(x) = 3x - 2$.
(v) $p(x) = 3x$
$p(x) = 0$
$3x = 0$
$x = 0$
Therefore, $x = 0$ is a zero of polynomial $p(x) = 3x$.
(vi) $p(x) = ax$
$p(x) = 0$
$ax = 0$
$x = 0$
Therefore, $x = 0$ is a zero of polynomial $p(x) = ax$.
(vii) $p(x) = cx + d$
$p(x) = 0$
$cx + d = 0$
$x = -d/c$
Therefore, $x = -d/c$ is a zero of polynomial $p(x) = cx + d$.
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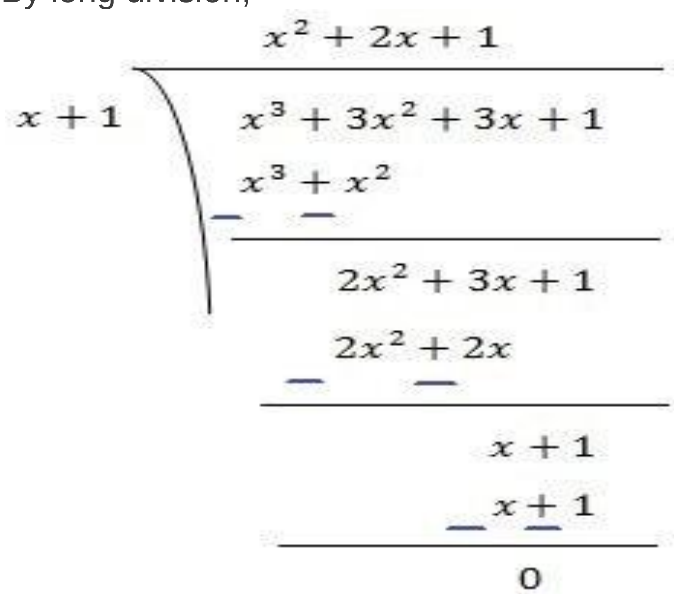
Exercises 2.3
1. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by
(i) $x + 1$
(ii) $x - 1/2$
(iii) x
(iv) $x + \pi$
(v) $5 + 2x$
Answer
(i) $x + 1$
By long division,



Therefore, the remainder is 0.

(ii) $x - 1/2$

By long division,

$$x^2 + \frac{7}{2}x + \frac{19}{4}$$

$$x - \frac{1}{2} \,\Big)\, x^3 + 3x^2 + 3x + 1$$

$$\underline{x^3 - \frac{x^2}{2}}$$
$$- \quad +$$

$$\frac{7}{2}x^2 + 3x + 1$$

$$\underline{\frac{7}{2}x^2 - \frac{7}{4}x}$$
$$- \quad +$$

$$\frac{19}{4}x + 1$$

$$\underline{\frac{19}{4}x - \frac{19}{8}}$$
$$- \quad +$$

$$\frac{27}{8}$$

Therefore, the remainder is 27/8.

(iii) x

$$x^2 + 3x + 3$$

$$x \,\Big)\, x^3 + 3x^2 + 3x + 1$$

$$\underline{x^3}$$
$$-$$

$$3x^2 + 3x + 1$$

$$\underline{3x^2}$$
$$-$$

$$3x + 1$$

$$\underline{3x}$$
$$-$$

$$1$$

Therefore, the remainder is 1.

(iv) $x + \pi$

$$x^2 + (3 - \pi)x + (3 - 3\pi + \pi^2$$

$$x + \pi \,\Big)\, x^3 + 3x^2 + 3x + 1$$

$$\underline{x^3 + \pi x^2}$$
$$- \quad -$$

$$(3 - \pi)x^2 + 3x + 1$$

$$\underline{(3 - \pi)x^2 + (3 - \pi)\pi x}$$
$$- \quad -$$

$$[3 - 3\pi + \pi^2]x + 1$$

$$\underline{[3 - 3\pi + \pi^2]x + (3 - 3\pi + \pi^2)\pi}$$
$$- \quad -$$

$$[1 - 3\pi + 3\pi^2 - \pi^3]$$

Therefore, the remainder is $[1 - 3\pi + 3\pi^2 - \pi^3]$.

(v) 5 + 2x

$$\begin{array}{r|l} & \frac{x^2}{2}+\frac{x}{4}+\frac{7}{8} \\ \hline 2x+5 & x^3+3x^2+3x+1 \\ & x^3+\frac{5}{2}x^2 \\ & -\quad - \\ \hline & \frac{x^2}{2}+3x+1 \\ & \frac{x^2}{2}+\frac{5x}{4} \\ & -\quad - \\ \hline & \frac{7x}{4}+1 \\ & \frac{7}{4}x+\frac{35}{8} \\ & -\quad - \\ \hline & -\frac{27}{8} \end{array}$$

Therefore, the remainder is -27/8.

2. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Answer

By Long Division,

$$\begin{array}{r|l} & x^2+6 \\ \hline x-a & x^3-ax^2+6x-a \\ & x^3-ax^2 \\ & -\quad + \\ \hline & 6x-a \\ & 6x-6a \\ & -\quad + \\ \hline & 5a \end{array}$$

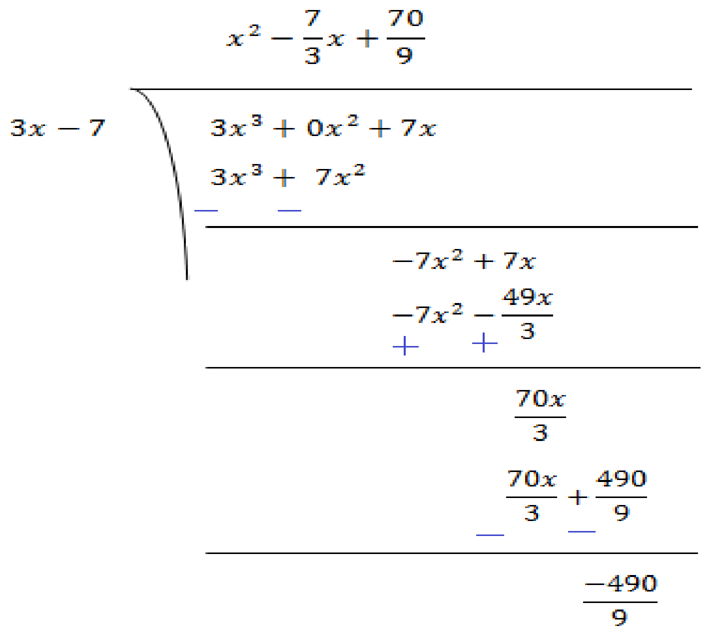
Therefore, remainder obtained is 5a when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

3. Check whether 7 + 3x is a factor of $3x^3 + 7x$.

Answer

We have to divide $3x^3 + 7x$ by $7 + 3x$. If remainder comes out to be 0 then $7 + 3x$ will be a factor of $3x^3 + 7x$.

By Long Division,



As remainder is not zero so $7 + 3x$ is not a factor of $3x^3 + 7x$.

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Exercise 2.4

1. Determine which of the following polynomials has $(x + 1)$ a factor:

(i) $x^3 + x^2 + x + 1$

(ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 – x^2 – (2 + \sqrt{2})x + \sqrt{2}$

Answer

(i) If $(x + 1)$ is a factor of $p(x) = x^3 + x^2 + x + 1$, $p(-1)$ must be zero.

Here, $p(x) = x^3 + x^2 + x + 1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 – 1 + 1 = 0$

Therefore, $x + 1$ is a factor of this polynomial

(ii) If $(x + 1)$ is a factor of $p(x) = x^4 + x^3 + x^2 + x + 1$, $p(-1)$ must be zero.

Here, $p(x) = x^4 + x^3 + x^2 + x + 1$

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 – 1 + 1 – 1 + 1 = 1$

As, $p(-1) \neq 0$

Therefore, $x + 1$ is not a factor of this polynomial

(iii)If $(x + 1)$ is a factor of polynomial $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$,$p(- 1)$ must be 0.

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 – 3 + 3 – 1 + 1 = 1$

As, $p(-1) \neq 0$

Therefore, $x + 1$ is not a factor of this polynomial.

(iv) If $(x + 1)$ is a factor of polynomial

$p(x) = x^3 – x^2 – (2 + \sqrt{2})x + \sqrt{2}$, $p(- 1)$ must be 0.

$p(-1) = (-1)^3 – (-1)^2 – (2 + \sqrt{2}) (-1) + \sqrt{2}$

$= -1 – 1 + 2 + \sqrt{2} + \sqrt{2}$

$=2\sqrt{2}$
As, $p(-1) \neq 0$
Therefore,, $x + 1$ is not a factor of this polynomial.
2. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:
(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$
(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$
(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$
Answer
(i) If $g(x) = x + 1$ is a factor of given polynomial $p(x)$, $p(-1)$ must be zero.
$p(x) = 2x^3 + x^2 - 2x - 1$
$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$
$= 2(-1) + 1 + 2 - 1 = 0$
Hence, $g(x) = x + 1$ is a factor of given polynomial.
(ii) If $g(x) = x + 2$ is a factor of given polynomial $p(x)$, $p(-2)$ must be 0.
$p(x) = x^3 + 3x^2 + 3x + 1$
$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$
$= -8 + 12 - 6 + 1$
$= -1$
As, $p(-2) \neq 0$
Hence $g(x) = x + 2$ is not a factor of given polynomial.
(iii) If $g(x) = x - 3$ is a factor of given polynomial $p(x)$, $p(3)$ must be 0.
$p(x) = x^3 - 4x^2 + x + 6$
$p(3) = (3)^3 - 4(3)^2 + 3 + 6$
$= 27 - 36 + 9 = 0$
Therefore,, $g(x) = x - 3$ is a factor of given polynomial.
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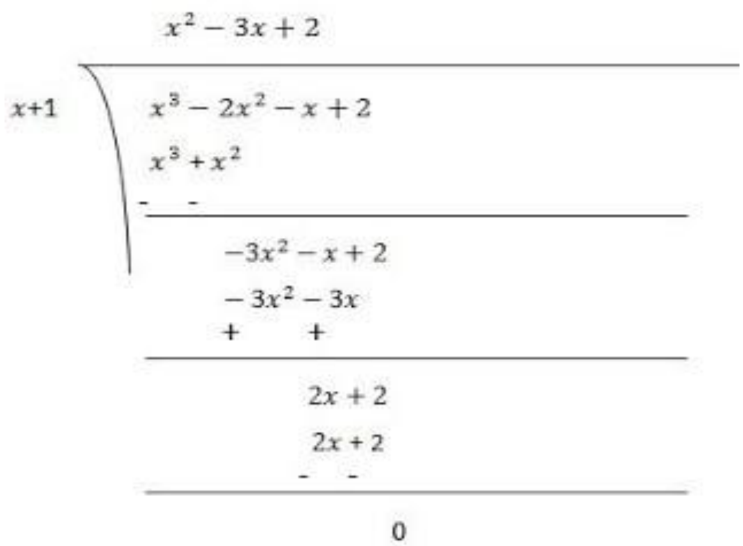
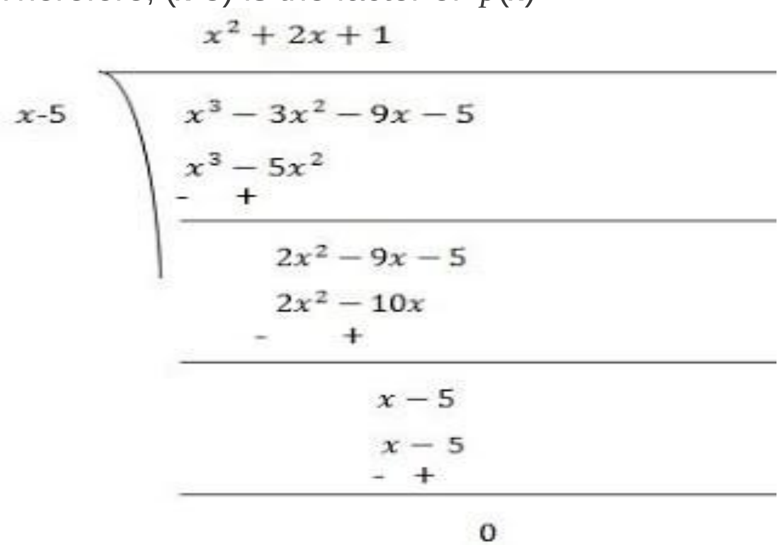
3. Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases:(i) $p(x) = x^2 + x + k$
(ii) $p(x) = 2x^2 + kx + \sqrt{2}$
(iii) $p(x) = kx^2 - \sqrt{2}x + 1$
(iv) $p(x) = kx^2 - 3x + k$
Answer
(i) If $x - 1$ is a factor of polynomial $p(x) = x^2 + x + k$, then
$p(1) = 0$
$\Rightarrow (1)^2 + 1 + k = 0$
$\Rightarrow 2 + k = 0$
$\Rightarrow k = -2$
Therefore, value of k is -2.
(ii) If $x - 1$ is a factor of polynomial $p(x) = 2x^2 + kx + \sqrt{2}$, then
$p(1) = 0$
$\Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0$
$\Rightarrow 2 + k + \sqrt{2} = 0$
$\Rightarrow k = -2 - \sqrt{2} = -(2 + \sqrt{2})$
Therefore, value of k is $-(2 + \sqrt{2})$.
(iii) If $x - 1$ is a factor of polynomial $p(x) = kx^2 - \sqrt{2}x + 1$, then
$p(1) = 0$
$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$

⇒ $k - \sqrt{2} + 1 = 0$
⇒ $k = \sqrt{2} - 1$
Therefore, value of k is $\sqrt{2} - 1$.
(iv) If $x - 1$ is a factor of polynomial $p(x) = kx^2 - 3x + k$, then
$p(1) = 0$
⇒ $k(1)^2 + 3(1) + k = 0$
⇒ $k - 3 + k = 0$
⇒ $2k - 3 = 0$
⇒ $k = 3/2$
Therefore, value of k is 3/2.
4. Factorise:
(i) $12x^2 + 7x + 1$
(ii) $2x^2 + 7x + 3$
(iii) $6x^2 + 5x - 6$
(iv) $3x^2 - x - 4$
Answer
(i) $12x^2 + 7x + 1$
$= 12x^2 - 4x - 3x + 1$
$= 4x (3x - 1) - 1 (3x - 1)$
$= (3x - 1) (4x - 1)$
(ii) $2x^2 + 7x + 3$
$= 2x^2 + 6x + x + 3$
$= 2x (x + 3) + 1 (x + 3)$
$= (x + 3) (2x + 1)$
(iii) $6x^2 + 5x - 6$
$= 6x^2 + 9x - 4x - 6$
$= 3x (2x + 3) - 2 (2x + 3)$
$= (2x + 3) (3x - 2)$
(iv) $3x^2 - x - 4$
$= 3x^2 - 4x + 3x - 4$
$= x (3x - 4) + 1 (3x - 4)$
$= (3x - 4) (x + 1)$
5. Factorise:
(i) $x^3 - 2x^2 - x + 2$
(ii) $x^3 - 3x^2 - 9x - 5$
(iii) $x^3 + 13x^2 + 32x + 20$
(iv) $2y^3 + y^2 - 2y - 1$
Answer
(i) Let $p(x) = x^3 - 2x^2 - x + 2$
Factors of 2 are ±1 and ± 2
By trial method, we find that
$p(1) = 0$
So, $(x+1)$ is factor of $p(x)$
Now,
$p(x) = x^3 - 2x^2 - x + 2$
$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2 = -1 -2 + 1 + 2 = 0$
Therefore, $(x+1)$ is the factor of $p(x)$



Now, Dividend = Divisor × Quotient + Remainder

$(x+1)\ (x^2 - 3x + 2)$
$= (x+1)\ (x^2 - x - 2x + 2)$
$= (x+1)\ \{x(x-1) - 2(x-1)\}$
$= (x+1)\ (x-1)\ (x+2)$
(ii) Let $p(x) = x^3 - 3x^2 - 9x - 5$
Factors of 5 are ±1 and ±5
By trial method, we find that
$p(5) = 0$
So, $(x-5)$ is factor of $p(x)$
Now,
$p(x) = x^3 - 2x^2 - x + 2$
$p(5) = (5)^3 - 3(5)^2 - 9(5) - 5 = 125 - 75 - 45 - 5 = 0$
Therefore, $(x-5)$ is the factor of $p(x)$



Now, Dividend = Divisor × Quotient + Remainder

$(x-5)\ (x^2 + 2x + 1)$
$= (x-5)\ (x^2 + x + x + 1)$
$= (x-5)\ \{x(x+1) + 1(x+1)\}$

$= (x-5)\ (x+1)\ (x+1)$

(iii) Let $p(x) = x^3 + 13x^2 + 32x + 20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

$p(-1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

$p(x) = x^3 + 13x^2 + 32x + 20$

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20 = -1 + 13 - 32 + 20 = 0$

Therefore, $(x+1)$ is the factor of $p(x)$

$$
\begin{array}{r|l}
 & x^2 + 12x + 20 \\
\hline
x+1 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & \quad 12x^2 + 32x + 20 \\
 & \quad 12x^2 + 12x \\
 & \quad - \quad - \\
\hline
 & \qquad 20x + 20 \\
 & \qquad 20x + 20 \\
 & \qquad - \quad - \\
\hline
 & \qquad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x+1)\ (x^2 + 12x + 20)$

$= (x+1)\ (x^2 + 2x + 10x + 20)$

$= (x-5)\ \{x(x+2) + 10(x+2)\}$

$= (x-5)\ (x+2)\ (x+10)$

(iv) Let $p(y) = 2y^3 + y^2 - 2y - 1$

Factors of ab = 2× (-1) = -2 are ±1 and ±2

By trial method, we find that

$p(1) = 0$

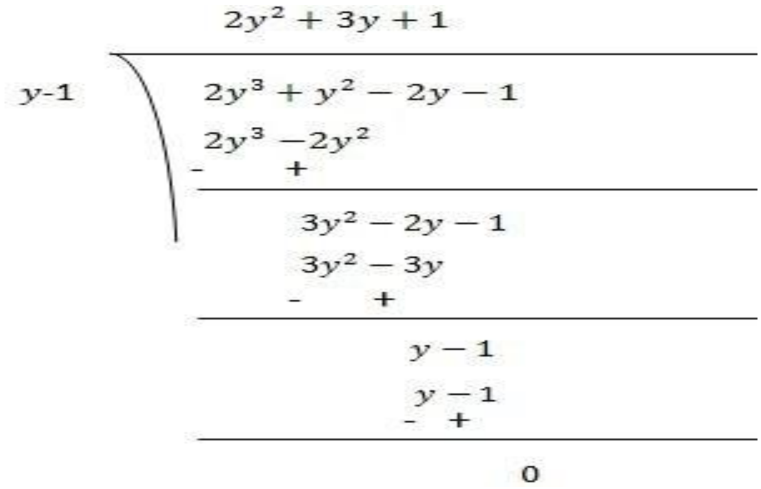
So, $(y-1)$ is factor of $p(y)$

Now,

$p(y) = 2y^3 + y^2 - 2y - 1$

$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1 = 2 + 1 - 2 - 1 = 0$

Therefore, $(y-1)$ is the factor of $p(y)$



Now, Dividend = Divisor × Quotient + Remainder

(y-1) $(2y^2 + 3y + 1)$
= (y-1) $(2y^2 + 2y + y + 1)$
= (y-1) {2y(y+1) +1(y+1)}
= (y-1) (2y+1) (y+1)
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Exercise 2.5
1. Use suitable identities to find the following products:
(i) $(x + 4) (x + 10)$ (ii) $(x + 8) (x – 10)$ (iii) $(3x + 4) (3x – 5)$
(iv) $(y^2 + 3/2) (y^2 – 3/2)$ (v) $(3 – 2x) (3 + 2x)$
Answer
(i) Using identity, $(x + a) (x + b) = x^2 + (a + b) x + ab$
In $(x + 4) (x + 10)$, a = 4 and b = 10
Now,
$(x + 4) (x + 10) = x^2 + (4 + 10)x + (4 × 10)$
$= x^2 + 14x + 40$

(ii) $(x + 8) (x – 10)$
Using identity, $(x + a) (x + b) = x^2 + (a + b) x + ab$
Here, a = 8 and b = –10
$(x + 8) (x – 10) = x^2 + \{8 +(– 10)\}x + \{8×(– 10)\}$
$= x^2 + (8 – 10)x – 80$
$= x^2 – 2x – 80$
(iii) $(3x + 4) (3x – 5)$
Using identity, $(x + a) (x + b) = x^2 + (a + b) x + ab$
Here, $x = 3x$, a = 4 and b = -5
$(3x + 4) (3x – 5) = (3x)^2 + \{4 + (-5)\}3x + \{4×(-5)\}$
$= 9x^2 + 3x(4 – 5) – 20$
$= 9x^2 – 3x – 20$
(iv) $(y^2 + 3/2) (y^2 – 3/2)$
Using identity, $(x + y) (x – y) = x^2 – y^2$
Here, $x = y^2$ and $y = 3/2$
$(y^2 + 3/2) (y^2 – 3/2) = (y^2)^2 – (3/2)^2$
$= y^4 – 9/4$

(v) $(3 - 2x)(3 + 2x)$
Using identity, $(x + y)(x - y) = x^2 - y^2$
Here, $x = 3$ and $y = 2x$
$(3 - 2x)(3 + 2x) = 3^2 - (2x)^2$
$= 9 - 4x^2$

2. Evaluate the following products without multiplying directly:
(i) 103 × 107 (ii) 95 × 96 (iii) 104 × 96

Answer
(i) 103 × 107 = (100 + 3) (100 + 7)
Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$
Here, $x = 100$, a = 3 and b = 7
103 × 107 = (100 + 3) (100 + 7) = $(100)^2$ + (3 + 7)10 + (3 × 7)
= 10000 + 100 + 21
= 11021
(ii) 95 × 96 = (90 + 5) (90 + 4)
Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$
Here, $x = 90$, a = 5 and b = 4
95 × 96 = (90 + 5) (90 + 4) = 90^2 + 90(5 + 6) + (5 × 6)
= 8100 + (11 × 90) + 30
= 8100 + 990 + 30 = 9120
(iii) 104 × 96 = (100 + 4) (100 – 4)
Using identity, $(x + y)(x - y) = x^2 - y^2$
Here, $x = 100$ and $y = 4$
104 × 96 = (100 + 4) (100 – 4) = $(100)^2 - (4)^2$ = 10000 – 16 = 9984

3. Factorise the following using appropriate identities:
(i) $9x^2 + 6xy + y^2$ (ii) $4y^2 - 4y + 1$ (iii) $x^2 - y^2/100$

Answer
(i) $9x^2 + 6xy + y^2 = (3x)^2 + (2\times 3x\times y) + y^2$
Using identity, $(a + b)^2 = a^2 + 2ab + b^2$
Here, a = $3x$ and b = y
$9x^2 + 6xy + y^2 = (3x)^2 + (2\times 3x\times y) + y^2 = (3x + y)^2 = (3x + y)(3x + y)$
(ii) $4y^2 - 4y + 1 = (2y)^2 - (2\times 2y\times 1) + 1^2$
Using identity, $(a - b)^2 = a^2 - 2ab + b^2$
Here, a = $2y$ and b = 1
$4y^2 - 4y + 1 = (2y)^2 - (2\times 2y\times 1) + 1^2 = (2y - 1)^2 = (2y - 1)(2y - 1)$
(iii) $x^2 - y^2/100 = x^2 - (y/10)^2$
Using identity, $a^2 - b^2 = (a + b)(a - b)$
Here, a = x and b = $(y/10)$
$x^2 - y^2/100 = x^2 - (y/10)^2 = (x - y/10)(x + y/10)$

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4. Expand each of the following, using suitable identities:
(i) $(x + 2y + 4z)^2$ (ii) $(2x - y + z)^2$ (iii) $(-2x + 3y + 2z)^2$
(iv) $(3a - 7b - c)^2$ (v) $(-2x + 5y - 3z)^2$ (vi) $[1/4\, a - 1/2\, b + 1]^2$

Answer

(i) $(x + 2y + 4z)^2$
Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
Here, $a = x$, $b = 2y$ and $c = 4z$
$(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + (2\times x\times 2y) + (2\times 2y\times 4z) + (2\times 4z\times x)$
$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$
(ii) $(2x - y + z)^2$
Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
Here, $a = 2x$, $b = -y$ and $c = z$
$(2x - y + z)^2 = (2x)^2 + (-y)^2 + z^2 + (2\times 2x\times -y) + (2\times -y\times z) + (2\times z\times 2x)$
$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$
(iii) $(-2x + 3y + 2z)^2$
Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
Here, $a = -2x$, $b = 3y$ and $c = 2z$
$(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + (2\times -2x\times 3y) + (2\times 3y\times 2z) + (2\times 2z\times -2x)$
$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$
(iv) $(3a - 7b - c)^2$
Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
Here, $a = 3a$, $b = -7b$ and $c = -c$
$(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + (2\times 3a\times -7b) + (2\times -7b\times -c) + (2\times -c\times 3a)$
$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$
(v) $(-2x + 5y - 3z)^2$
Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
Here, $a = -2x$, $b = 5y$ and $c = -3z$
$(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + (2\times -2x\times 5y) + (2\times 5y\times -3z) + (2\times -3z\times -2x)$
$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$
(vi) $[1/4\, a - 1/2\, b + 1]^2$
Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
Here, $a = 1/4\, a$, $b = -1/2\, b$ and $c = 1$
$[1/4\, a - 1/2\, b + 1]^2 = (1/4\, a)^2 + (-1/2\, b)^2 + 1^2 + (2\times 1/4\, a\times -1/2\, b) + (2\times -1/2\, b\times 1) + (2\times 1\times 1/4\, a)$
$= 1/16\, a^2 + 1/4\, b^2 + 1 - 1/4\, ab - b + 1/2\, a$
5. Factorise:
(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$
(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\, xy + 4\sqrt{2}\, yz - 8xz$

Answer

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$
Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$
$= (2x)^2 + (3y)^2 + (-4z)^2 + (2\times 2x\times 3y) + (2\times 3y\times -4z) + (2\times -4z\times 2x)$
$= (2x + 3y - 4z)^2$
$= (2x + 3y - 4z)(2x + 3y - 4z)$
(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\, xy + 4\sqrt{2}\, yz - 8xz$
Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
$2x^2 + y^2 + 8z^2 - 2\sqrt{2}\, xy + 4\sqrt{2}\, yz - 8xz$
$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + (2\times -\sqrt{2}x\times y) + (2\times y\times 2\sqrt{2}z) + (2\times 2\sqrt{2}z\times -\sqrt{2}x)$
$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$
$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$

6. Write the following cubes in expanded form:

(i) $(2x + 1)^3$ (ii) $(2a - 3b)^3$ (iii) $[3/2\ x + 1]^3$ (iv) $[x - 2/3\ y]^3$

Answer

(i) $(2x + 1)^3$

Using identity, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$(2x + 1)^3 = (2x)^3 + 1^3 + (3\times2x\times1)(2x + 1)$

$= 8x^3 + 1 + 6x(2x + 1)$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a - 3b)^3$

Using identity, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$(2a - 3b)^3 = (2a)^3 - (3b)^3 - (3\times2a\times3b)(2a - 3b)$

$= 8a^3 - 27b^3 - 18ab(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $[3/2\ x + 1]^3$

Using identity, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$[3/2\ x + 1]^3 = (3/2\ x)^3 + 1^3 + (3\times3/2\ x\times1)(3/2\ x + 1)$

$= 27/8\ x^3 + 1 + 9/2\ x(3/2\ x + 1)$

$= 27/8\ x^3 + 1 + 27/4\ x^2 + 9/2\ x$

$= 27/8\ x^3 + 27/4\ x^2 + 9/2\ x + 1$

(iv) $[x - 2/3\ y]^3$

Using identity, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$[x - 2/3\ y]^3 = (x)^3 - (2/3\ y)^3 - (3\times x\times2/3\ y)(x - 2/3\ y)$

$= x^3 - 8/27y^3 - 2xy(x - 2/3\ y)$

$= x^3 - 8/27y^3 - 2x^2y + 4/3xy^2$

7. Evaluate the following using suitable identities:

(i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$

Answer

(i) $(99)^3 = (100 - 1)^3$

Using identity, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$(100 - 1)^3 = (100)^3 - 1^3 - (3\times100\times1)(100 - 1)$

$= 1000000 - 1 - 300(100 - 1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $(102)^3 = (100 + 2)^3$

Using identity, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$(100 + 2)^3 = (100)^3 + 2^3 + (3\times100\times2)(100 + 2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Using identity, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$(1000 - 2)^3 = (1000)^3 - 2^3 - (3\times1000\times2)(1000 - 2)$

$= 100000000 - 8 - 6000(1000 - 2)$

$= 100000000 - 8- 600000 + 12000$

$= 994011992$

8. Factorise each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$ (ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$ (iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - 1/216 - 9/2\ p^2 + 1/4\ p$

Answer

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

Using identity, $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + b^3 + 3(2a)^2b + 3(2a)(b)^2$

$= (2a + b)^3$

$= (2a + b)(2a + b)(2a + b)$

(ii) $8a^3 – b^3 – 12a^2b + 6ab^2$

Using identity, $(a – b)^3 = a^3 – b^3 – 3a^2b + 3ab^2$

$8a^3 – b^3 – 12a^2b + 6ab^2 = (2a)^3 – b^3 – 3(2a)^2b + 3(2a)(b)^2$

$= (2a – b)^3$

$= (2a – b)(2a – b)(2a – b)$

(iii) $27 – 125a^3 – 135a + 225a^2$

Using identity, $(a – b)^3 = a^3 – b^3 – 3a^2b + 3ab^2$

$27 – 125a^3 – 135a + 225a^2 = 3^3 – (5a)^3 – 3(3)^2(5a) + 3(3)(5a)^2$

$= (3 – 5a)^3$

$= (3 – 5a)(3 – 5a)(3 – 5a)$

(iv) $64a^3 – 27b^3 – 144a^2b + 108ab^2$

Using identity, $(a – b)^3 = a^3 – b^3 – 3a^2b + 3ab^2$

$64a^3 – 27b^3 – 144a^2b + 108ab^2 = (4a)^3 – (3b)^3 – 3(4a)^2(3b) + 3(4a)(3b)^2$

$= (4a – 3b)^3$

$= (4a – 3b)(4a – 3b)(4a – 3b)$

(v) $27p^3 – 1/216 – 9/2\ p^2 + 1/4\ p$

Using identity, $(a – b)^3 = a^3 – b^3 – 3a^2b + 3ab^2$

$27p^3 – 1/216 – 9/2\ p^2 + 1/4\ p$

$= (3p)^3 – (1/6)^3 – 3(3p)^2(1/6) + 3(3p)(1/6)^2$

$= (3p – 1/6)^3$

$= (3p – 1/6)(3p – 1/6)(3p – 1/6)$

9. Verify : (i) $x^3 + y^3 = (x + y) (x^2 – xy + y^2)$ (ii) $x^3 – y^3 = (x – y) (x^2 + xy + y^2)$

Answer

(i) $x^3 + y^3 = (x + y) (x^2 – xy + y^2)$

We know that,

$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow x^3 + y^3 = (x + y)^3 – 3xy(x + y)$

$\Rightarrow x^3 + y^3 = (x + y)[(x + y)^2 – 3xy]$ *{Taking (x+y) common}*

$\Rightarrow x^3 + y^3 = (x + y)[(x^2 + y^2 + 2xy) – 3xy]$

$\Rightarrow x^3 + y^3 = (x + y)(x^2 + y^2 – xy)$

(ii) $x^3 – y^3 = (x – y) (x^2 + xy + y^2)$

We know that,

$(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$\Rightarrow x^3 – y^3 = (x – y)^3 + 3xy(x – y)$

$\Rightarrow x^3 + y^3 = (x – y)[(x – y)^2 + 3xy]$ *{Taking (x-y) common}*

$\Rightarrow x^3 + y^3 = (x – y)[(x^2 + y^2 – 2xy) + 3xy]$

$\Rightarrow x^3 + y^3 = (x + y)(x^2 + y^2 + xy)$

10. Factorise each of the following:

(i) $27y^3 + 125z^3$ (ii) $64m^3 – 343n^3$

Answer

(i) $27y^3 + 125z^3$

Using identity, $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$= (3y + 5z)\{(3y)^2 - (3y)(5z) + (5z)^2\}$

$= (3y + 5z)(9y^2 - 15yz + 25z)^2$

(ii) $64m^3 - 343n^3$

Using identity, $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$64m^3 - 343n^3 = (4m)^3 - (7n)^3$

$= (4m + 7n)\{(4m)^2 + (4m)(7n) + (7n)^2\}$

$= (4m + 7n)(16m^2 + 28mn + 49n)^2$

11. Factorise : $27x^3 + y^3 + z^3 - 9xyz$

Answer

$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3\times3xyz$

Using identity, $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$27x^3 + y^3 + z^3 - 9xyz$

$= (3x + y + z)\{(3x)^2 + y^2 + z^2 - 3xy - yz - 3xz\}$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

12. Verify that: $x^3 + y^3 + z^3 - 3xyz = 1/2(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Answer

We know that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 1/2\times(x + y + z)\,2(x^2 + y^2 + z^2 - xy - yz - xz)$

$= 1/2(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz)$

$= 1/2(x + y + z)[(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (x^2 + z^2 - 2xz)]$

$= 1/2(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Answer

We know that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

Now put $(x + y + z) = 0$,

$x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - xz)$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 0$

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Answer

(i) $(-12)^3 + (7)^3 + (5)^3$

Let $x = -12$, $y = 7$ and $z = 5$

We observed that, $x + y + z = -12 + 7 + 5 = 0$

We know that if,

$x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

$(-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5) = -1260$

(ii) $(28)^3 + (–15)^3 + (-13)^3$
Let $x = 28$, $y = -15$ and $z = -13$
We observed that, $x + y + z = 28 – 15 – 13 = 0$

We know that if,
$x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$
$(28)^3 + (–15)^3 + (-13)^3 = 3(28)(-15)(-13) = 16380$
15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:
(i) Area : $25a^2 – 35a + 12$
(ii) Area : $35y^2 + 13y – 12$

Answer
(i) Area : $25a^2 – 35a + 12$

Since, area is product of length and breadth therefore by factorizing the given area, we can know the length and breadth of rectangle.
$25a^2 – 35a + 12$
$= 25a^2 – 15a - 20a + 12$
$= 5a(5a – 3) – 4(5a – 3)$
$= (5a – 4)(5a – 3)$
Possible expression for length $= 5a – 4$
Possible expression for breadth $= 5a – 3$
(ii) Area : $35y^2 + 13y – 12$
$35y^2 + 13y – 12$
$= 35y^2 – 15y + 28y – 12$
$= 5y(7y – 3) + 4(7y – 3)$
$= (5y + 4)(7y – 3)$
Possible expression for length $= (5y + 4)$
Possible expression for breadth $= (7y – 3)$
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16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below? (i) Volume : $3x^2 – 12x$
(ii) Volume : $12ky^2 + 8ky – 20k$
Answer
(i) Volume : $3x^2 – 12x$
Since, volume is product of length, breadth and height therefore by factorizing the given volume, we can know the length, breadth and height of the cuboid.
$3x^2 – 12x$
$= 3x(x – 4)$
Possible expression for length $= 3$
Possible expression for breadth $= x$
Possible expression for height $= (x – 4)$
(ii) Volume : $12ky^2 + 8ky – 20k$
Since, volume is product of length, breadth and height therefore by factorizing the given volume, we can know the length, breadth and height of the cuboid.
$12ky^2 + 8ky – 20k$
$= 4k(3y^2 + 2y – 5)$

$= 4k(3y^2 + 5y – 3y – 5)$
$= 4k[y(3y + 5) – 1(3y + 5)]$
$= 4k (3y + 5) (y – 1)$
Possible expression for length = $4k$
Possible expression for breadth = $(3y + 5)$
Possible expression for height = $(y – 1)$

NCERT Solutions for Class IX Maths: Chapter 3 – Coordinate Geometry

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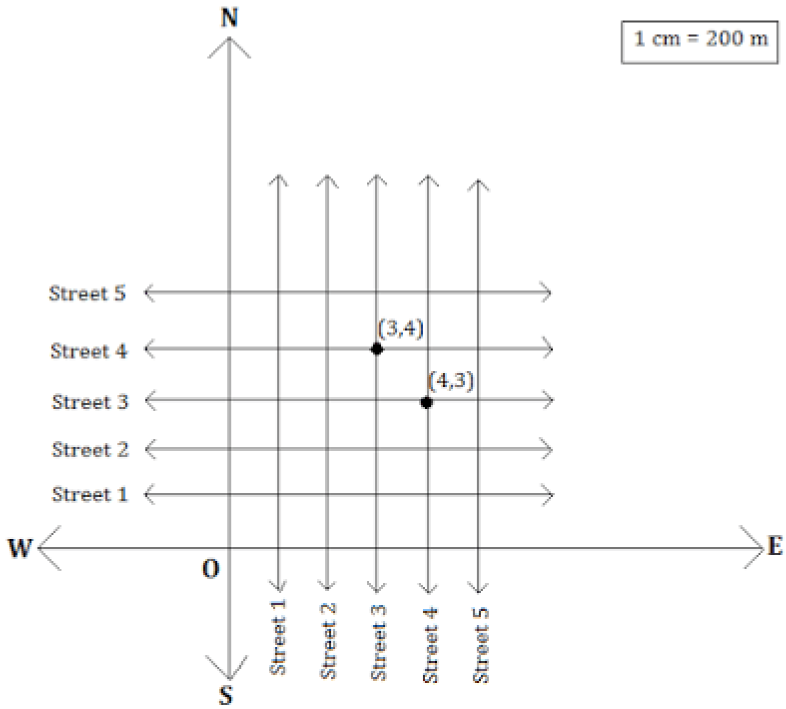
Exercise 3.1
1. How will you describe the position of a table lamp on your study table to another person?

Answer
To describe the position of a table lamp on the study table, we have two take two lines, a perpendicular and horizontal. Considering the table as a plane and taking perpendicular line as Y axis and horizontal as X axis. Take one corner of table as origin where both X and Y axes intersect each other. Now, the length of table is Y axis and breadth is X axis. From The origin, join the line to the lamp and mark a point. Calculate the distance of this point from both X and Y axes and then write it in terms of coordinates.
Let the distance of point from X axis is x and from Y axis is y then the the position of the table lamp in terms of coordinates is (x,y).

2. (Street Plan) : A city has two main roads which cross each other at the centre of the city. These two roads are along the North-South direction and East-West direction.
All the other streets of the city run parallel to these roads and are 200 m apart. There are 5 streets in each direction. Using 1cm = 200 m, draw a model of the city on your notebook. Represent the roads/streets by single lines. There are many cross- streets in your model. A particular cross-street is made by two streets, one running in the North – South direction and another in the East – West direction. Each cross street is referred to in the following manner : If the 2nd street running in the North – South direction and 5 th in the East – West direction meet at some crossing, then we will call this cross-street (2, 5). Using this convention, find:
(i) how many cross – streets can be referred to as (4, 3).
(ii) how many cross – streets can be referred to as (3, 4)

Answer



(i) Only one street can be referred to as (4, 3) as we see from the figure.

(ii) Only one street can be referred to as (3, 4) as we see from the figure.

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Exercise 3.2
1. Write the answer of each of the following questions:

(i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?
(ii) What is the name of each part of the plane formed by these two lines?
(iii) Write the name of the point where these two lines intersect.

Answer
(i) The name of horizontal lines and vertical lines drawn to determine the position of any point in the Cartesian plane is x-axis and y-axis respectively.

(ii) The name of each part of the plane formed by these two lines x-axis and y-axis is quadrants.

(iii) The point where these two lines intersect is called origin.

2. See Fig.3.14, and write the following:

(i) The coordinates of B.
(ii) The coordinates of C.
(iii) The point identified by the coordinates (-3, -5).
(iv) The point identified by the coordinates (2, -4).
(v) The abscissa of the point D.
(vi) The ordinate of the point H.

(vii)The coordinates of the point L.
(viii) The coordinates of the point M.

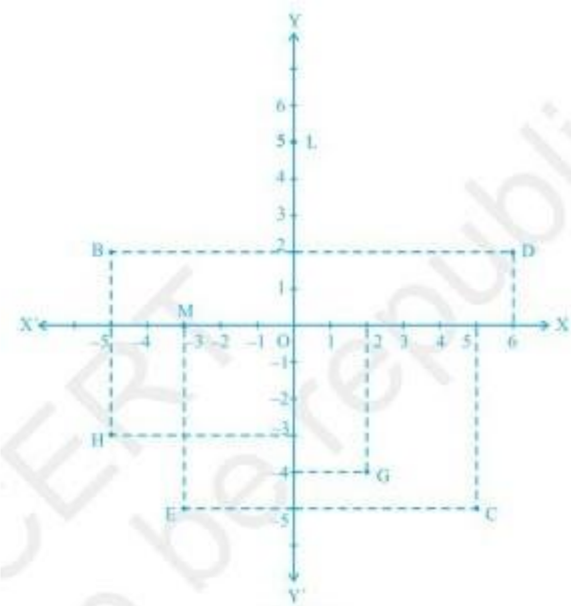

Fig. 3.14

Answer
(i) The coordinates of B is (-5, 2).

(ii) The coordinates of C is (5, -5).

(iii) The point identified by the coordinates (-3, -5) is E.

(iv) The point identified by the coordinates (2, -4) is G.

(v) Abscissa means x coordinate of point D. So, abscissa of the point D is 6.

(vi) Ordinate means y coordinate of point H. So, ordinate of point H is -3.

(vii) The coordinates of the point L is (0, 5).

(viii) The coordinates of the point M is (- 3, 0).

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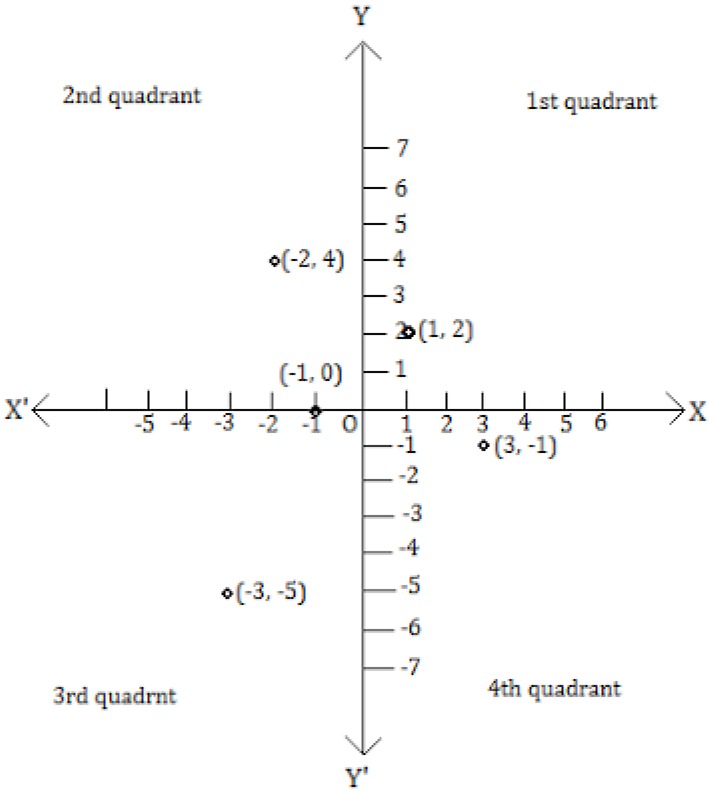
Exercise 3.3
1. In which quadrant or on which axis do each of the points (-2, 4), (3, -1), (-1, 0), (1, 2) and (-3, -5) lie? Verify your answer by locating them on the Cartesian plane.

Answer

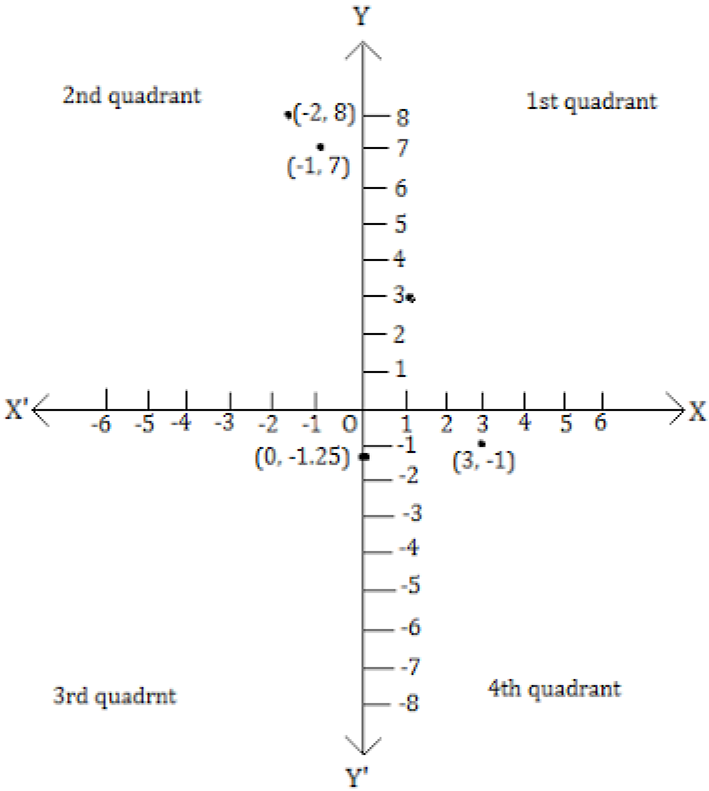


(-2, 4) → Second quadrant
(3, -1) → Fourth quadrant
(-1, 0) → Second quadrant
(1, 2) → First quadrant
(-3, -5) → Third quadrant

2. Plot the points (x, y) given in the following table on the plane, choosing suitable units of distance on the axes.

x	-2	-1	0	1
y	8	7	-1.25	3

Answer
Points (x,y) on the plane. 1unit = 1 cm


Y
2nd quadrant
1st quadrant
(-2, 8)
(-1, 7)
8
7
6
5
4
3
2
1
X'
X
-6 -5 -4 -3 -2 -1 O 1 2 3 4 5 6
(0, -1.25)
(3, -1)
-1
-2
-3
-4
-5
-6
-7
-8
3rd quadrnt
4th quadrant
Y'